Review: Fourier Transform and Modulation

Wednesday, October 29, 2014 12:51 PM

Fourier Transform

Time domain | Freq. domain

Fourier transform

$$g(t) = \int_{-\infty}^{\infty} G(f)\underbrace{e^{j2\pi f t}}_{\text{complex exponential}}\, df \quad \underset{\mathcal{F}^{-1}}{\overset{\mathcal{F}}{\rightleftarrows}} \quad G(f) = \int_{-\infty}^{\infty} g(t)e^{-j2\pi f t}\, dt$$

inverse Fourier transform

$$e^{j2\pi f t} = \cos(2\pi f t) + j\sin(2\pi f t)$$

$$g(0) = \int_{-\infty}^{\infty} G(f)\, df \qquad G(0) = \int_{-\infty}^{\infty} g(t)\, dt$$

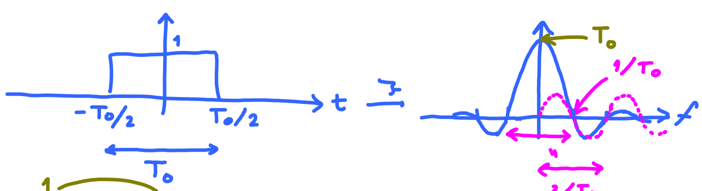
Important transform pairs:

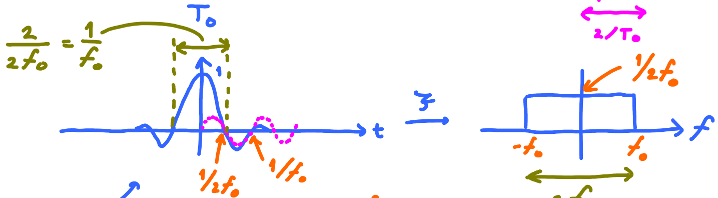
$$e^{j2\pi f_0 t} \xrightarrow{\mathcal{F}} \delta(f - f_0)$$

$$1 \xrightarrow{\mathcal{F}} \delta(f)$$

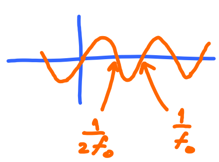
$$\delta(t) \xrightarrow{\mathcal{F}} 1$$

rectangular function $\xrightarrow{\mathcal{F}}$ sinc function $\qquad \text{sinc}\, x = \frac{\sin x}{x}$





$$\text{sinc}(2\pi f_0 t) = \frac{\sin(2\pi f_0 t)}{2\pi f_0 t}$$



Important Properties

$$\int_{-\infty}^{\infty} x(\tau)y(t-\tau)d\tau = x(t) * y(t) \xrightarrow{\mathcal{F}} X(f)Y(f)$$

$$x(t)y(t) \xrightarrow{\mathcal{F}} X(f) * Y(f)$$

$$g(t)e^{j2\pi f_0 t} \xrightarrow{\mathcal{F}} G(f) * \delta(f - f_0) = G(f - f_0)$$

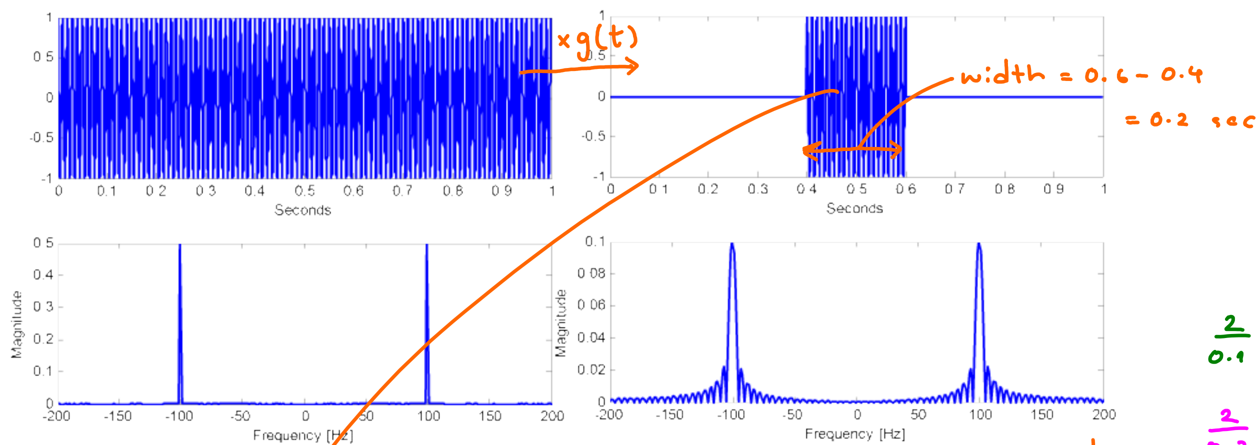
$$x(t)\,y(t) \xrightarrow{\mathcal{F}} X(f) * Y(f)$$

$$g(t)\,e^{j2\pi f_0 t} \xrightarrow{\mathcal{F}} G(f) * \delta(f-f_0) = G(f-f_0)$$

$$m(t)\cos(2\pi f_0 t) = m(t)\times \frac{e^{j2\pi f_0 t} + e^{j2\pi(-f_0)t}}{2} \xrightarrow{\mathcal{F}} \frac{1}{2}M(f-f_0) + \frac{1}{2}M(f-(-f_0))$$

$$\left.\begin{aligned} e^{jx} &= \cos x + j\sin x \\ e^{-jx} &= \cos x - j\sin x \end{aligned}\right\}\quad e^{jx}+e^{-jx} = 2\cos x$$

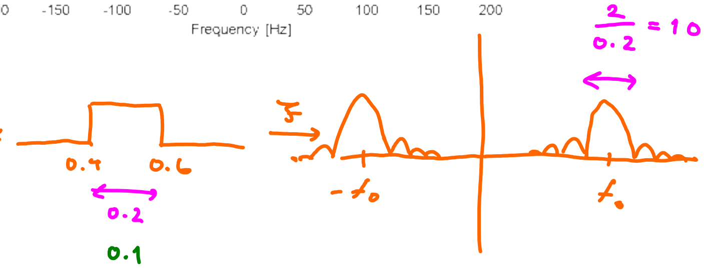
$$x(t) = \cos(2\pi(100)t) \qquad x(t) = \begin{cases} \cos(2\pi(100)t), & 0.4 \le t \le 0.6, \\ 0, & \textit{otherwise}. \end{cases}$$



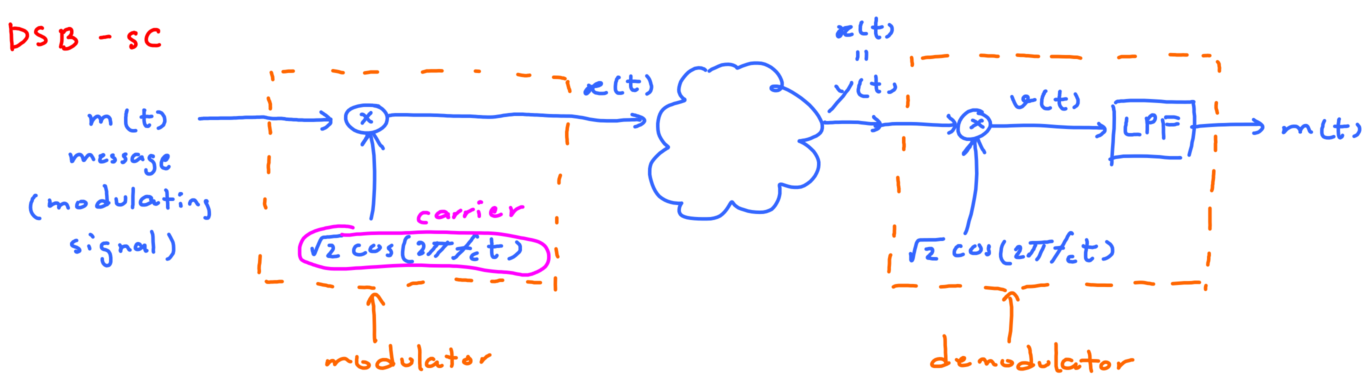
$\frac{2}{0.1} = 20$

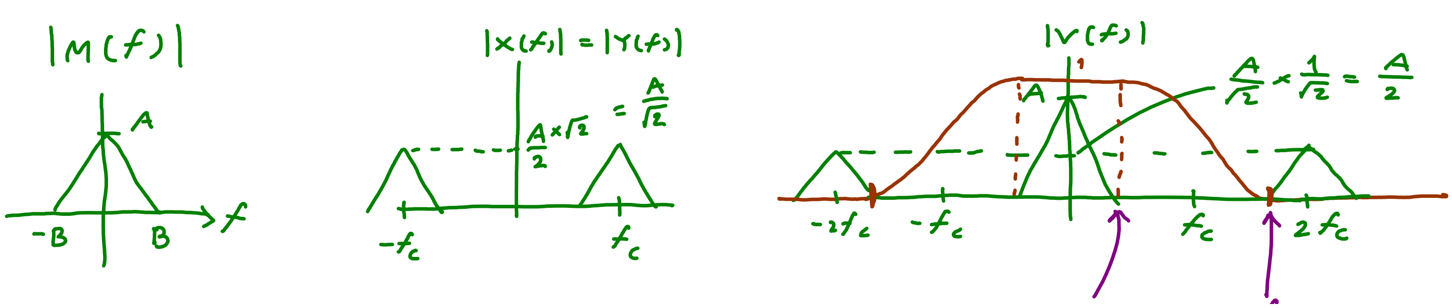
$\frac{2}{0.2} = 10$

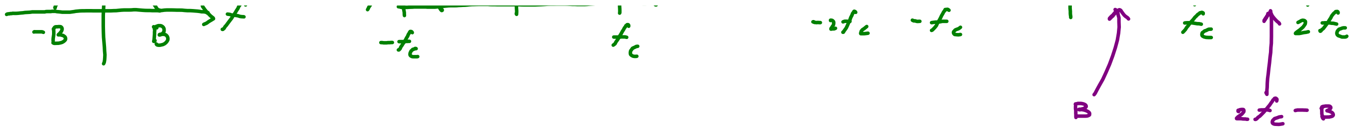
cosine pulse = $\cos(2\pi f_0 t)\times$



DSB - SC







Assumption : $2f_c - B > B$

$f_c > B$

Key equation : $\text{LPF}\left\{ \underbrace{\left(\underbrace{m(t)\sqrt{2}\cos(2\pi f_c t)}_{x(t)} \right) \times \sqrt{2}\cos(2\pi f_c t)}_{v(t)} \right\} = m(t).$

Alternatively,

$$v(t) = m(t)\, \underbrace{2\cos^2(2\pi f_c t)} = m(t)\left(1 + \cos(2\pi(2f_c)t)\right)$$

$$= m(t) + m(t)\cos(2\pi(2f_c)t)$$

$$e^{-j2\pi f k T_s} = \cos(2\pi f k T_s) - j\sin(2\pi f k T_s)$$

$$= \cos(2\pi(kT_s)f) - j\sin(2\pi(kT_s)f)$$

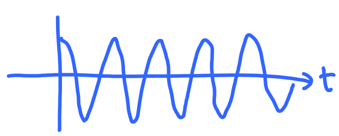
$\cos(2\pi 5 x)$

$\cos(2\pi 5 x + \theta)$

$= \cos\left(2\pi 5\left(x + \frac{\theta}{2\pi 5}\right)\right)$

freq = 5 cycles per unit of x

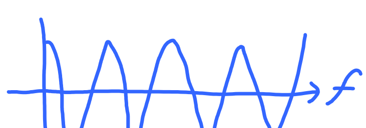
$\cos(2\pi 5 t)$



freq = 5 cycles per unit of t
time

$\underbrace{\qquad\qquad}_{\text{Hz}}$

$\cos(2\pi 5 f)$



"freq" = 5 cycles per unit of f

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More properties of Fourier transform

① Scaling property : $g(at) \xrightarrow{\mathcal{F}} \frac{1}{|a|} G\left(\frac{f}{a}\right)$

② Duality theorem: $G(t) \xrightarrow{\mathcal{F}} g(-f)$

$\left(g(t) \xrightarrow{\mathcal{F}} G(f) \right)$

③ Parseval's theorem

$$\langle g_1, g_2 \rangle = \langle G_1, G_2 \rangle$$

$$\int_{-\infty}^{\infty} g_1(t) g_2^*(t)\, dt = \int_{-\infty}^{\infty} G_1(f) G_2^*(f)\, df$$

Fourier transform preserves the inner product.

$$\text{Energy of } g(t) \equiv \int_{-\infty}^{\infty} |g(t)|^2 dt = \int_{-\infty}^{\infty} \underbrace{|G(f)|^2}_{\text{Energy spectral density}} df$$